EFFECT OF PHASE ANGLE ON THE EFFICIENCY OF BETA TYPE STIRLING ENGINE

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Abstract

A Stirling Engine is a mechanical device, which operates on a closed regenerative cycle, based on cyclic compression and expansion through a piston and a displacer respectively. It can be widely used for many thermodynamic purposes such as stationary power generation, heat pumps or co-generation systems. Due to the external supply of heat and increasing scope of solar energy utility in Pakistan, this engine can be operated successfully with this useful source of energy. Phase angle is an important parameter of the Stirling engine and is one of the key factors on which performance of the engine depends. It is the angle by which expansion space volume leads the compression space with respect to the volume variations in the engine cycle. This paper describes the optimization and modelling of the phase angle of a single cylinder beta Stirling Engine with Helium as the working fluid. Schmidt analysis is considered to be the standard during this research for analysing the output efficiency of the engine. The volume and pressure variations are computed at different values of phase angle for a complete cycle and ultimately values chart and pressure-volume diagrams are prepared. The work done for each case is calculated for finding the optimum phase angle. It is calculated that the suitable phase angle for the maximum efficiency of the engine is around 90. Along with maximum and minimum pressure inside the engine, the overlap volume in beta type Stirling engine plays a vital role and efficiency increases with increase in overlap region, however, this volume is limited to engine geometry and displacer and piston timing.
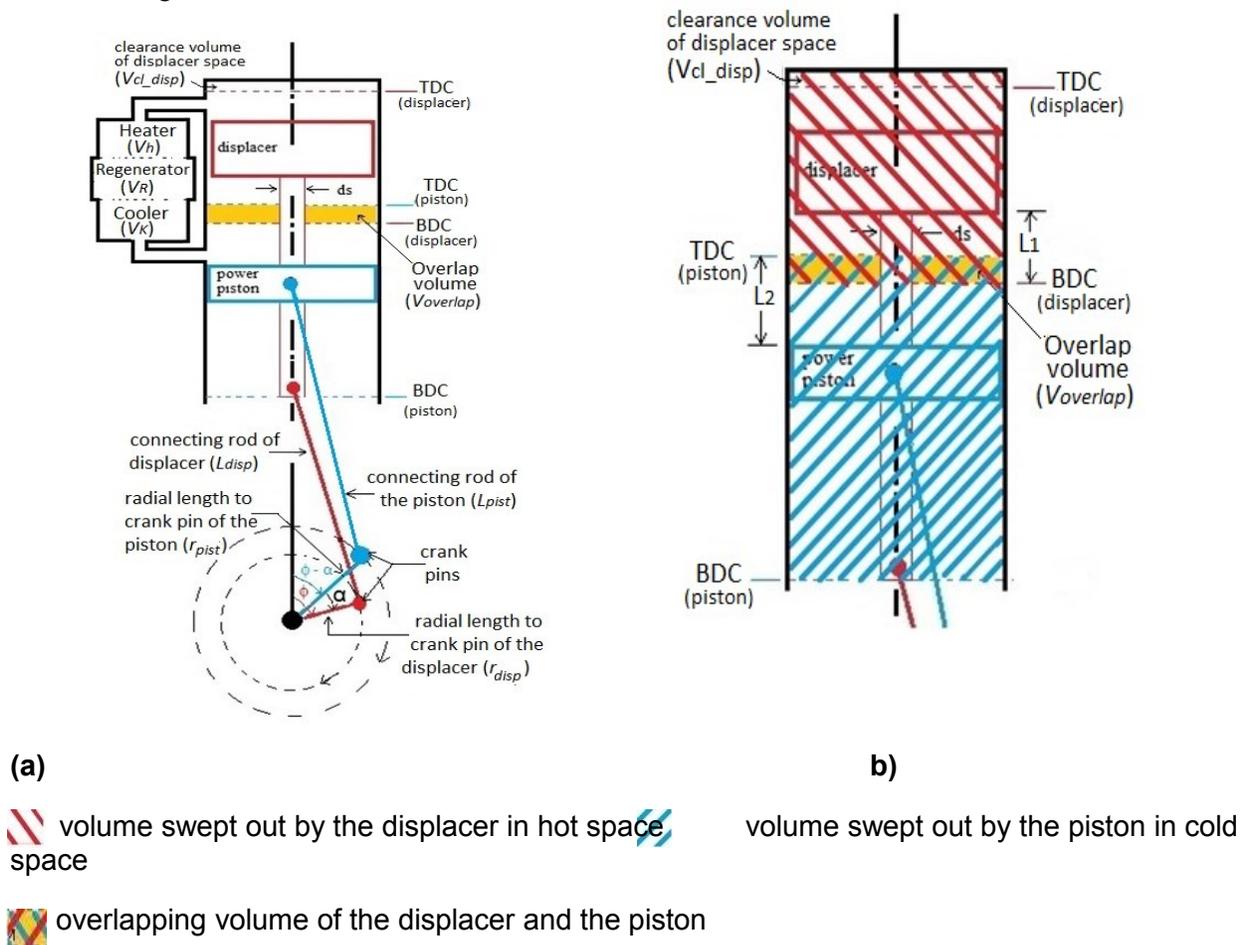
Keywords:
Beta Stirling engine, phase angle, work done, overlap volume, instantaneous pressure

1. Introduction

The use of the natural sources of energy is increasing day by day due their availability and the economical approach. The research is in progress on natural and renewable sources of energy in all parts of the world [1], [2] including the conversion of natural energy to mechanical energy[3] with increasing significance of solar energy [4], [5]. This has increased the importance of the plants and the engines operated on the external supply of heat. A

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Stirling engine is an external combustion engine which operates on closed regenerative cycle. The first hot air engine of this kind was built in 1816 by Rev. Robert Stirling in Scotland. The working fluid may be air or another gas (normally hydrogen or helium) with a difference in temperature levels in such a way that there is a net conversion of heat to work or vice versa. The term 'regenerative' in Stirling engines refers to the use of a specific type of internal heat exchanger and thermal store known as the regenerator. Stirling engines are primarily preferred in applications where the use of air-breathing engines is not feasible like in submarines and space. The energy sources are varied to suit the requirements and can be solar, chemical or even nuclear. The classical analysis of the operation of the Stirling engines was derived by Schmidt in 1871 [6]. Much of the research, afterwards, has been conducted for designing the Stirling engines [7]. There are three configurations of the Stirling engine namely alpha, beta and gamma. In comparison to other two configurations, the beta type has a single cylinder in which both the displacer and the working piston are mounted [8]. The most important and unique characteristic of beta type engine is that displacer and piston may share a part of their swept volumes known as overlap volume which is an important part of the current research. This is shown in Fig. 1.

(a)   **b)**

◥◥ volume swept out by the displacer in hot space, ⫽⫽ volume swept out by the piston in cold space

▨▨ overlapping volume of the displacer and the piston

Fig. 1: Beta type Stirling engine with its volumetric configuration

In beta type Stirling engines (Fig.1); there is a displacer piston along with a single power piston. Both of the pistons are arranged within the same cylinder. The displacer piston does not require a good seal and it is not meant for extracting any power from the expanding gas, rather, it serves to shuttle the working gas between the hot and cold ends of the cylinder. However,

Eid [9] used the displacer space of a beta Stirling engine occupied by wire meshes for successful application of regeneration. Similarly, Karabulut et al. [10] claimed better approximation to theoretical Stirling cycle by means of lever controlled displacer driving mechanism and augmentation of displacer space by growth of span wise slots. The working gas is pushed to the hot end of the cylinder by the displacer, and heat is supplied to the cycle at a high temperature during expansion of the working fluid. Work is done, by the working fluid, equal in magnitude to the heat supplied and pushing the displacer to the bottom dead centre (bdc). For an ideal cycle, there is no change in the internal energy, but an increase in the entropy of the working fluid. The force requirement on the displacer is very small as in a well-designed Stirling engine, the pressure drop across the regenerator and heat exchangers will be small, and the area of the displacer facing downward is only slightly smaller than the area facing upwards. When moving up, the displacer shuttles the gas towards the cold end of the cylinder through regenerator. The heat is abstracted from the working fluid into the regenerator and heat is subsequently rejected from the cycle at the minimum cycle temperature as the power piston compresses the gas. Work is done on the working fluid equal in magnitude to the heat rejected from the cycle. For an ideal cycle, there is no change in internal energy, but there is a decrease in entropy. Unlike the alpha type engine, the beta type avoids the technical problems of hot moving seals on the power piston. A pseudo Stirling engine model with temperature profile is shown in Fig. 2.
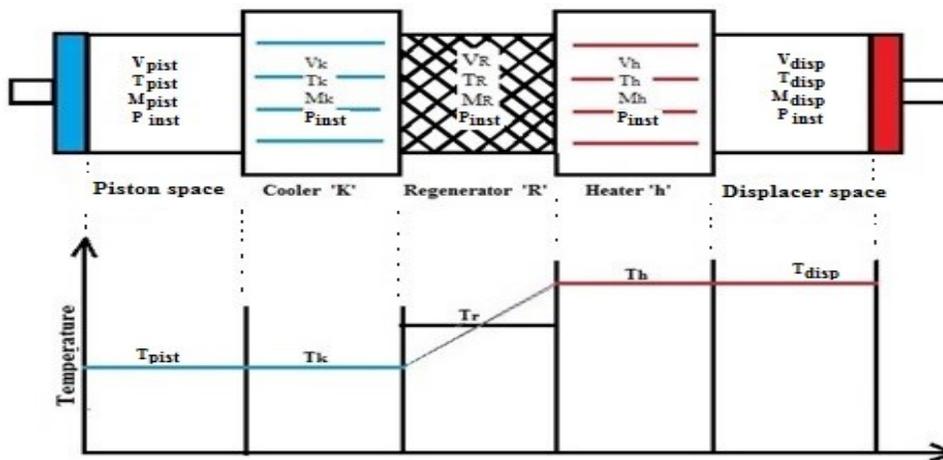

Fig. 2 Pseudo Stirling engine model

Kirkley[11].introduced a dimensionless quantity called power parameter for optimizing the Stirling engine performance. The present research is based on the optimization of a small beta type Stirling engine with a particular reference to phase angle (ϕ) through simulations and modelling which are useful for further experimental work. The phase angle principally affects the volume variations and is one of the key factors upon which efficiency of the engine depends. The current research would be useful for the analysis of efficiency of the engine at different values of phase angle with a combination of other factors in order to observe how desired efficiency can be achieved through optimum value of phase angle.

2. Mathematical calculations

2.1. Volumetric analysis

In order to have the volumetric analysis of the beta type Stirling engine, the total volume of the engine is divided into five major parts as shown in Fig.2 for any instantaneous moment with

reference to the volume of the gas present in expansion space, compression space, heater, regenerator and cooler while volumes of the dead spaces and clearance volumes are also taken in account. As per Schmidt analysis [6], the volume of the displacer space above the displacer at any instance (V_{inst_disp}) is given by:

$$V_{inst\_disp} = (A_{disp}) [(r_{disp}) (1- cos\phi) + L_{disp} (1 - \sqrt{1 - \frac{r_{disp}^2}{L_{disp}^2} sin^2\phi})] + V_{D\_disp} + V_{cl\text{-}disp} \quad ---(1)$$

Where, A_{disp} and V_{D_disp} are the area of the displacer and dead volume in the displacer space respectively. In Fig. 1, the volumetric configuration is illustrated on the basis of displacer and piston space. The instantaneous volume of the piston space (V_{inst_pist}) in a beta type Stirling engine is the volume between the displacer and the piston at any moment. Since, area of the displacer at the bottom side is less than that of the area on the top side because of presence of displacer rod at the bottom with diameter 'ds' as shown in Fig. 1. If 'r_disp_rod' and 'A_{disp_rod}' are the 'radius' and 'area' of the displacer rod respectively, then

$$r\_disp\_rod = \frac{1}{2}(ds) \quad \text{or} \quad A_{disp\_rod} = \pi (r\_disp\_rod)^2 \quad\quad ---(2)$$

Volume between instantaneous position of displacer and its bottom dead centre (represented by the part of stroke length as L_1)

$$= (A_{disp} - A_{disp\_rod}) [(r_{disp}) (1+ cos\phi) - L_{disp} (1 - \sqrt{1 - \frac{r_{disp}^2}{L_{disp}^2} sin^2\phi})] \quad ----(3)$$

Volume between instantaneous position of piston and its top dead centre (represented by the part of stroke length as L_2)

$$= (A_{pist}) \{ (r_{pist}) [1- cos(\phi - \alpha)] + L_{pist} [1 - \sqrt{1 - \frac{r_{pist}^2}{L_{pist}^2} sin^2(\phi - \alpha)}]\} \quad ----(4)$$

The instantaneous volume of the piston space (V_{inst_pist}) is calculated by subtracting overlapping volume ($V_{overlap}$) of displacer and piston spaces from the sum of the volume between the displacer and its bottom dead centre (represented by the part of stroke length as L_1) and the volume between piston and its top dead centre (represented by the part of stroke length as L_2), considering the clearance and dead volumes in account. Mathematically,

$$V_{inst\_pist} = (A_{disp} - A_{disp\_rod}) [(r_{disp}) (1+ cos\phi) - L_{disp}(1 - \sqrt{1 - \frac{r_{disp}^2}{L_{disp}^2} sin^2\phi})] + (A_{pist}) (r_{pist}) \{[1-$$

$$cos(\phi - \alpha)] + L_{pist} [1 - \sqrt{1 - \frac{r_{pist}^2}{L_{pist}^2} sin^2(\phi - \alpha)}]\} - V_{overlap} + V_{D\_pis} - V_{cl\text{-}pist} \quad ----(5)$$

Where, '$V_{cl\text{-}pist}$' is the clearance volume of the power piston movement with respect to the displacer movement. The minimum clearance along with overlap volume and the angle 'α' is illustrated in figure 3 with sinusoidal movements of displacer and power piston.
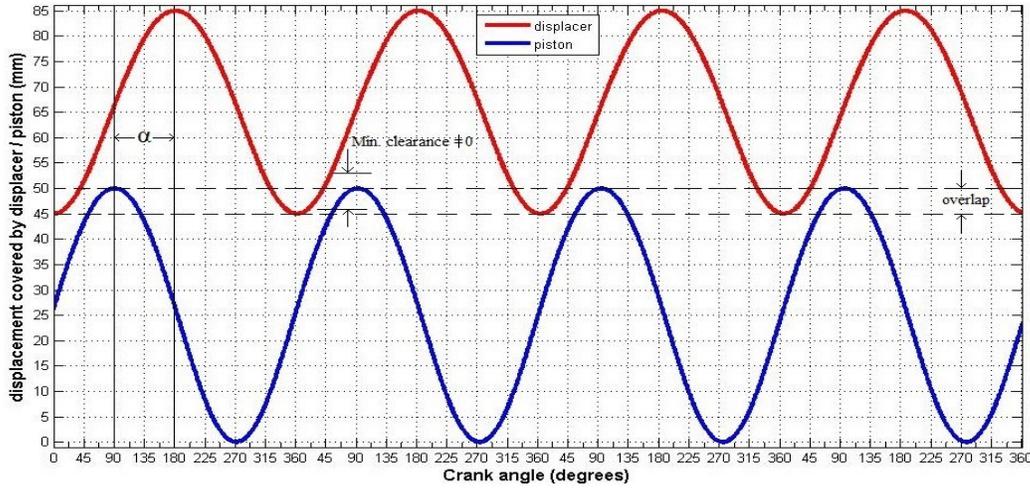
Fig. 3 Sinusoidal movement of the displacer and the power piston in a beta Stirling engine

The total instantaneous volume (*V*) of the engine is calculated to be as:

$$V = \quad V_{inst\_disp} + V_{inst\_pist} + V_h + V_R + V_K \qquad\qquad \text{----(6)}$$

Where, V_h, V_R and V_K are the volume of hot space, volume of regenerator and volume of cold space respectively

$$V = \{(A_{disp})\,[\,(r_{disp})\,(1-\cos\phi) + L_{disp}\,(\,1 - \sqrt{1 - \frac{r_{disp}^2}{L_{disp}^2}\sin^2\phi}\,)\,] + V_{D\_disp} + V_{cl\text{-}disp}\}$$

$$+ (A_{disp} - A_{disp\_rod})\,\{\,(r_{disp})\,(1+\cos\phi) - L_{disp}\,(\,1 - \sqrt{1 - \frac{r_{disp}^2}{L_{disp}^2}\sin^2\phi}\,)\,\}$$

$$+ (A_{pist})\,\{\,(r_{pist})\,[1 - \cos(\phi\text{-}\alpha)] + L_{pist}\,[\,1 - \sqrt{1 - \frac{r_{pist}^2}{L_{pist}^2}\sin^2(\phi-\alpha)}\,] - V_{overlap}$$

$$+ V_{D\_pist} - V_{cl\text{-}pist}\} + V_h + V_R + V_K \qquad\qquad \text{------- (7)}$$

where, for sinusoidal motion,

$$V_{overlap} = \frac{V_{sw\_disp} + V_{sw\_pist}}{2} - \sqrt{\frac{(V_{sw\_disp})^2 + (V_{sw\_pist})^2}{4} - \frac{(V_{sw\_disp})\,(V_{sw\_pist})}{2}\cos(\alpha)} \quad \text{------- (8)}$$

2.2. Pressure Analysis

Bancha and Somchai [12] found mean pressure and Beale formula appropriate for calculation of Stirling engine power. For the model of Stirling engine as shown in Fig.2, at start, total mass of the working gas (M_T) is constant and is sum of the mass present in all component areas.

$$M_T = M_{disp} + M_{pist} + M_h + M_R + M_K \quad\quad \text{------- (9)}$$

If T_{disp}, T_{pist}, T_h, T_R, and T_K represent the temperatures in displacer space, piston space, heater, regenerator and cooler respectively and R is the universal gas constant, by Universal Gas law:

$$M_T = \frac{P_{disp}V_{inst\_disp}}{R\,T_{disp}} + \frac{P_{pist}\,V_{inst\_pist}}{R\,T_{pist}} + \frac{P_h\,V_h}{R\,T_h} + \frac{P_R\,V_R}{R\,T_R} + \frac{P_K V_K}{R\,T_K} \quad \text{---- (10)}$$

Since, the instantaneous pressure (P_{inst}) is assumed to be the same throughout the system, so,

$$M_T = \frac{P_{inst}}{R}\left(\frac{V_{inst\_disp}}{T_{disp}} + \frac{V_{inst\_pist}}{T_{pist}} + \frac{V_h}{T_h} + \frac{V_R}{T_R} + \frac{V_K}{T_K}\right) \quad\quad \text{------- (11)}$$

Since, $\quad\quad\quad\quad\quad\quad T_{disp} = T_h \quad \& \quad T_{pist} = T_K$

Eq (11) can be written as:

$$P_{inst} = \frac{M_T\,R}{\left(\frac{V_{inst\_pist}}{T_K} + \frac{V_K}{T_K} + \frac{V_R}{T_R} + \frac{V_h}{T_h} + \frac{V_{inst\_disp}}{T_h}\right)} \quad\quad \text{------- (12)}$$

If Lr is the length of the regenerator and 'x' is the distance of a certain point from the cooler, the temperature profile of the regenerator will be:
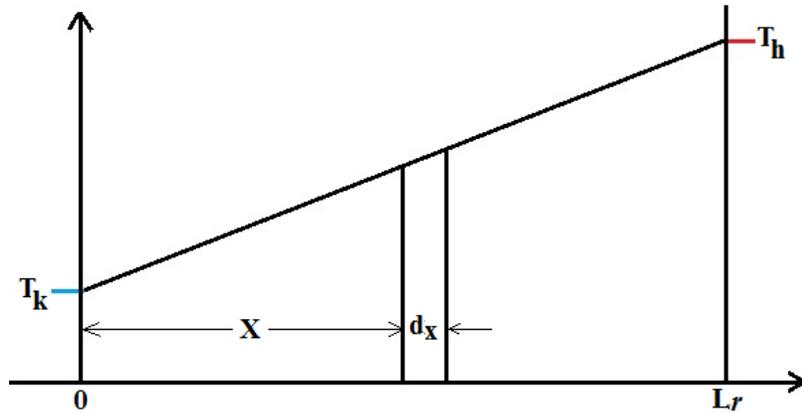

Fig. 4 Regenerator linear temperature profile

By the calculations, the temperature profile of the regenerator, Eq. 12 can be expressed as:

$$P_{inst} = \frac{M_T\,R}{\left[\frac{V_{inst\,pist}}{T_K} + \frac{V_K}{T_K} + \frac{V_R\,ln\left(\frac{Th}{Tk}\right)}{(Th-Tk)} + \frac{V_h}{T_h} + \frac{V_{inst\,disp}}{T_h}\right]} \quad\quad \text{------- (13)}$$

Eq. (13) describes the pressure of the gas in the engine at any instance.

2.3. Calculation of Work done

In order to calculate the work done (W), the following expression is used:

$$W = \sum \left[(V_{i+1} - V_i) * \left(\frac{P_{i+1} + P_i}{2} \right) \right] \qquad \text{------- (14)}$$

3. Results and discussion

The specifications of the small beta engine are described in Table-I. Most of these parameters are not able to be changed, however some of these can be altered accordingly.

Table–I: Specifications of the beta type Stirling engine

Specifications	Size (cm^3)
Swept volume of the displacer	113.14
Swept volume of the piston space	140.52
Volume of the regenerator	7.48
Volume of the cooler	26.43
Volume of the heater	9.75
Volume of overlap region	14.05
Volume of min. clearance of piston and displacer	23.09
Clearance volume of the displacer space	16.97
Dead volume of the displacer space	33.83

A *matlab* programme was set in order to calculate the optimum conditions at different values of phase angle in accordance with the specifications mentioned in Table-I. The working gas inside the engine is considered to be Helium and the temperature in the hot and cold space was taken to be 900 K and 300 K respectively.
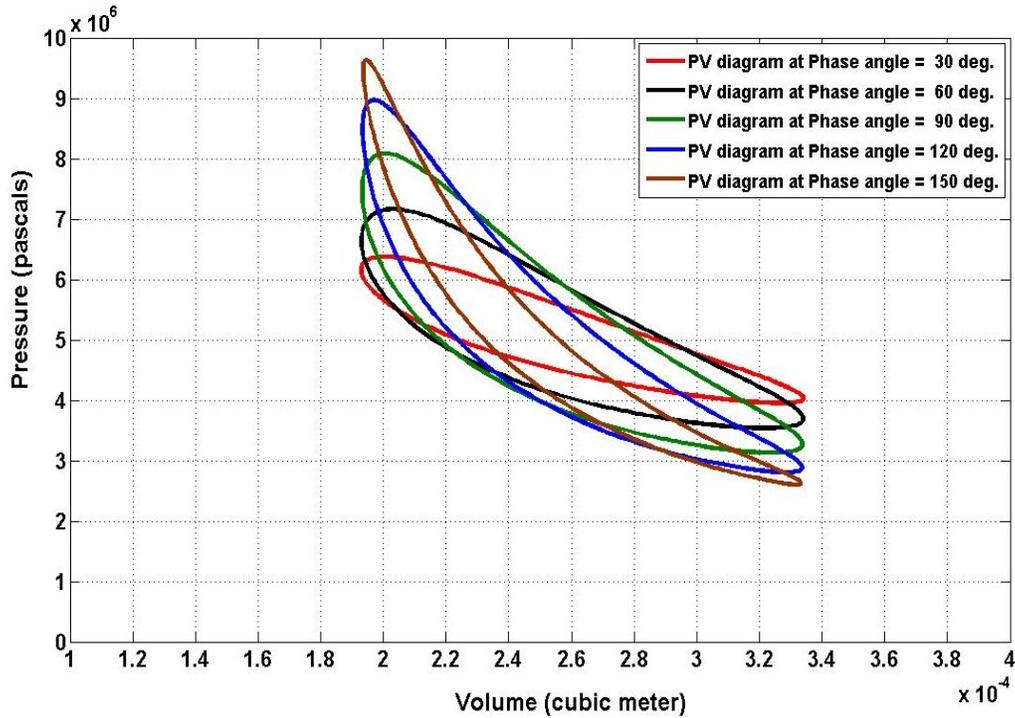
3.1. Work done

Based on the calculations of instantaneous pressure and the volume for a cycle, work done is calculated and Pressure vs Volume (PV) diagram is constructed for specified values of phase angle shown in Fig. 5.

The PV-diagram in Fig. 5 indicates the work done at different phase angles ranging from 30° to 150°. It reveals that the PV diagram for the phase angle of 90° has the widest area and hence the maximum work will be achieved when the engine would run with the phase angle around 90° (±5°). With the set program in matlab and generated spread sheets, the value of work done by the beta engine was calculated for phase angle values range increasing from 1° to 180°. The results are illustrated graphically in Fig. 6.

From Fig. 6, it is revealed that upon increasing the piston phase angle from 1° to 180°, the value of output work increases gradually and then decreases in the same way until it becomes *zero* at

phase angle 180°. The value of phase angle may change ±5° from 90° as it depends upon the combination of different other parameters the values of which may change with change in conditions. In the present case, the peak value of the work done is found to be 241.05 Joules.


Fig. 5 Pressure/Volume Graph of beta Stirling Cycle
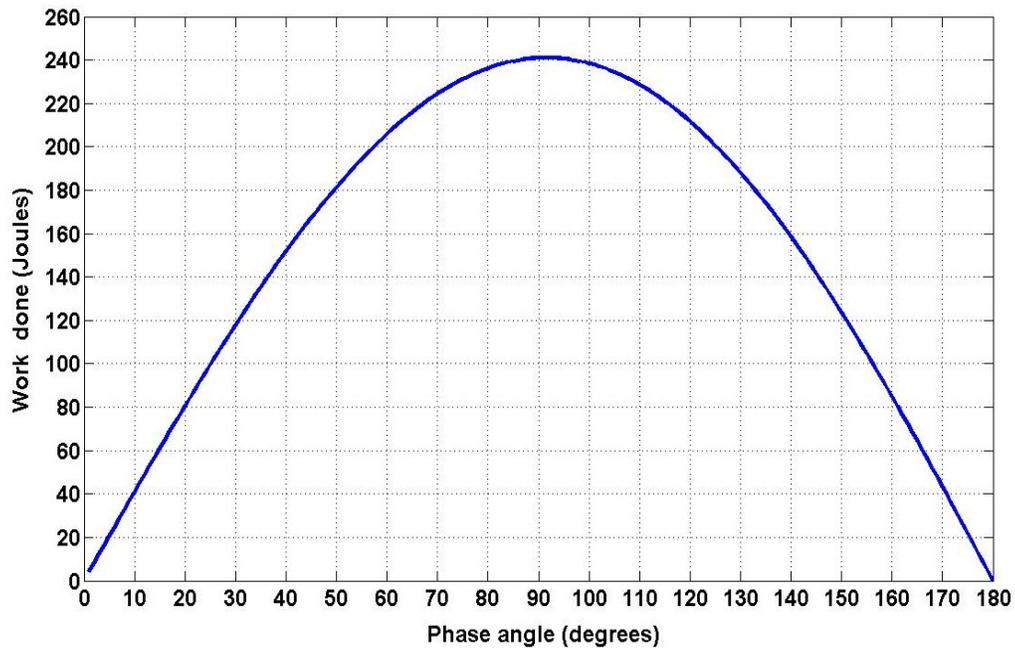
Fig. 6 Graph between the phase angle and the Work-done in a beta type Stirling engine

3.2. Overlap volume

Overlap volume is the volume shared by both displacer and the piston space. It is difficult to change the value of overlap volume in an engine practically; however by altering the position of connecting rod in slider crank mechanism may affect the overlap volume. In the present case the overlap volume of the beta type Stirling engine is 14.05 cm^3. In order to analyse the effect of overlapping region in the engine and to attain the maximum output, different values of overlap volume are selected to analyse its effect on the work done for different phase angles. In this regard, contour plot is drawn from the values in the spread sheet which is illustrated in Fig. 7. The plot reveals that the amount of work done is the combination of the
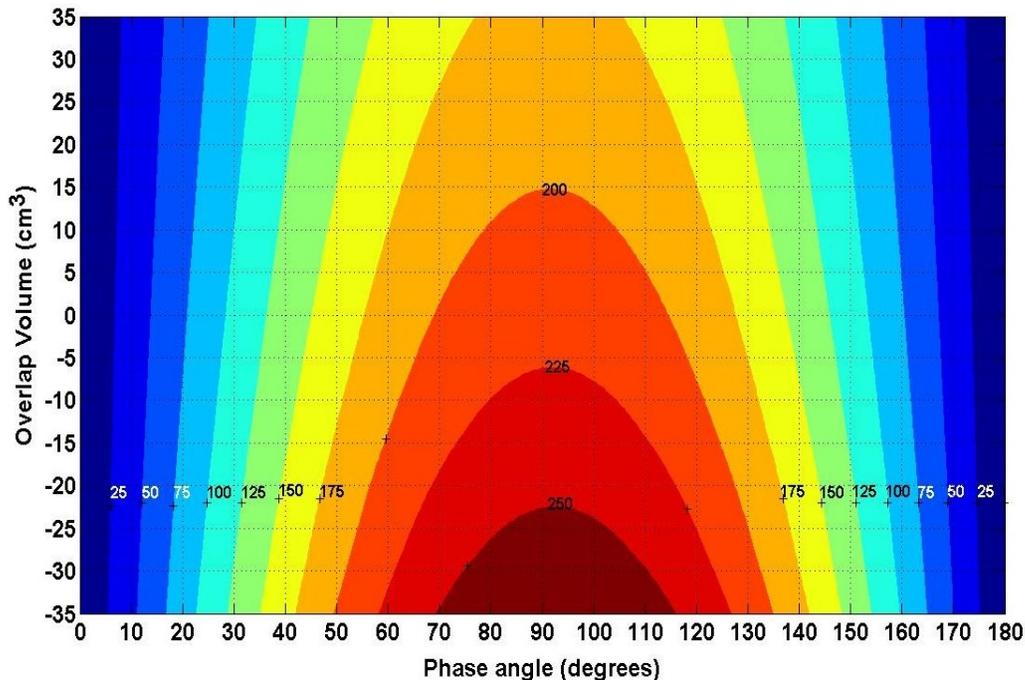

Fig. 7 Contour plot: Work done against Phase angle and Overlap volume in Beta Stirling engine

phase angle and the overlap volume in this case. The required amount of work can be achieved with a suitable combination of overlap volume (of the displacer and the piston space) and the phase angle. The negative value of the overlap volume of the displacer and the piston demonstrate that the displacer and the piston share that volume in their motion and is the volume between top dead centre of piston and bottom dead centre of displacer while the positive value indicates that volume is not used by either the displacer or the piston It is notable that higher values of work done are achieved with maximum overlapping region. However, it is pertinent to mention that there is still a limit to this overlapping as the timing of the displacer and the piston is also adjusted, otherwise the displacer and the piston may collide with each other. The adjustment of timing primarily depends upon the phase angle as through which expansion and the compression space variations take place with the movement of displacer and the piston accordingly.

3.3. Maximum and minimum instantaneous pressure

The pressure in the engine at any moment is regarded as the instantaneous pressure and maximum and minim limit is needs to be defined. It is notable that the maximum limit is quite important as the capacity of the engine to bear the pressure is taken in regard. For the beta type Stirling engine, the value of 'maximum instantaneous pressure' is found to be increased gradually with the increase in phase angle throughout the test as shown in Fig. 8 (a).

(a)   (b)
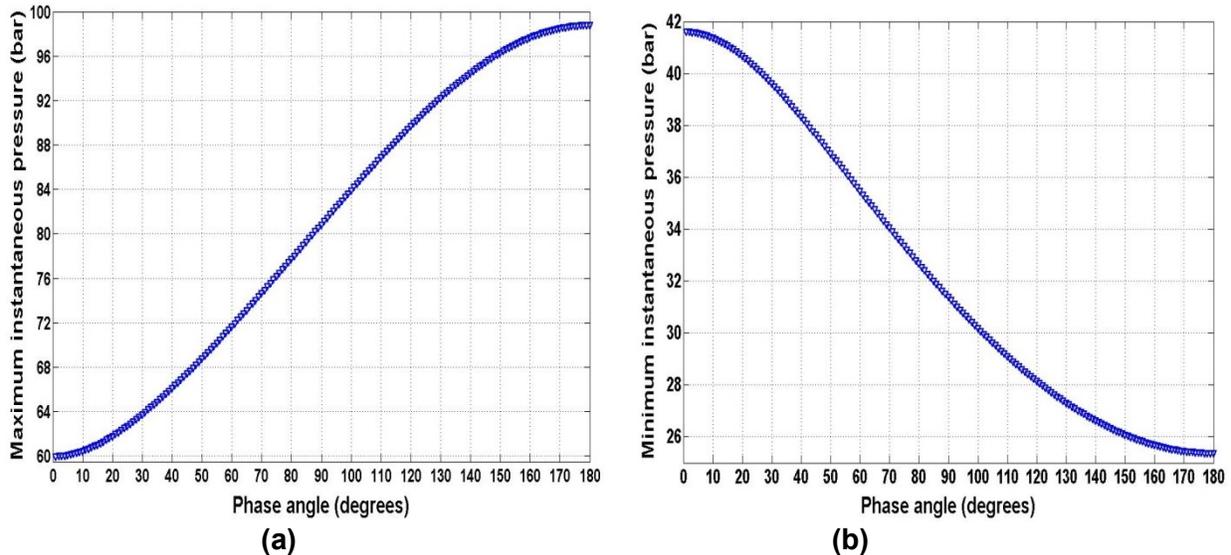
Fig. 8 Graph between the phase angle and instantaneous pressure (max. & min) in beta Stirling engine

The value of 'minimum instantaneous pressure' is found to be decreasing steadily throughout as the phase angle increases. This is graphically illustrated in 8 (b). By choosing the suitable combination of both extends of instantaneous pressure vales, optimization is done. For the said engine, the output mean pressure is set to be at 50 bars in every combination.

4.   Conclusions

Developmental efforts on Stirling engines indicate substantial ability for future applications. Due to its simplicity and low cost, reliability factor is quite promising. The phase angle plays a vital role in Stirling engine performance. It was found that for the maximum efficiency, the phase angle was found to be 90°. The efficiency decreases gradually as the phase angle is increased or decreased from its optimum value and becomes minimum at 0° and 180° respectively.

Overlap volume, which is a unique characteristic of beta type Stirling engine needs optimization to attain maximum output efficiency. In the present case, the overlap volume (14.05 cm^3) in combination of the optimum value of phase angle concludes the work done to be 236.12 Joules. It was also revealed that increasing the overlap volume results in increase in engine efficiency.

It was also noted that the phase angle affects the instantaneous pressure limit. Increasing the phase angle, the maximum instantaneous pressure gradually increases. Similarly, the value of minimum instantaneous pressure decreases gradually with decrease in phase angle. These values are very important in designing an engine in order to keep in mind the capacity of the engine to bear pressure. For the present case, the value of minimum instantaneous pressure at

optimum phase angle was found to be 31.32 bar while the value of maximum instantaneous pressure appeared to be 81.45 bar.

However, the safety factor is needed to be taken in account in this regard focussing the capacity of the engine to withstand with the extreme conditions. For instance, the engine body and the material should have capability to withstand the extreme pressure limits. Similarly, the importance of the temperature bearing range of the material in the expansion space should also be kept in mind. In the same way, the safety limits regarding other parameters may also be considered. Selecting the optimized values of the aforementioned parameters promises the desired output of the beta type Stirling engine keeping safety factor in account.

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